Using LaTeX to generate dynamic mathematics worksheets for the web

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Abstract

Mathematics worksheet generators abound on the web. Many use static content and focus on graphics and animation in order to package the material in an appealing manner. This approach comes across as a fight for *eyeballs* — all too common when trying to attract the target audience on the Internet. The emphasis on form often displaces the basis of learning at the primary education level, which is simple practice. Beginning with an exploration of effective learning strategies for grade school mathematics, the use of LaTeX to generate dynamic mathematics worksheets — lots and lots of them — is discussed.

1 Introduction

The impetus of this project came from witnessing my daughter's struggles with mastering maths in early grades. The teachers — well intentioned as they were — felt that she was getting bogged down in mastering basic numeracy and that once freed from this mechanical process, she would be able to progress more quickly. Unfortunately, the evidence was to the contrary: she continued to struggle, and moreover slipped further and further behind.

In this paper, I discuss what I believe to be the importance of *numeracy* as a necessary step in achieving mathematical literacy, and the value of its co-conspirator, *practice*, which is the essential method of learning for young students.

The reader would be right in criticizing the paper's lack of statistical evidence to support the conclusions, and may in fact consider these conclusions to be mere conjectures. To this I plead *mea culpa*. I hope that a reasoned argument will suffice for the time being, and perhaps encourage others to take the exploration further. In my defence, however, I did not seek out to prove the correctness of my thesis; I was simply searching for techniques to help my daughter achieve tangible results.

2 The decline of practice

It is my belief that like for any skill, practice is at the heart of mastering elementary school mathematics. However, in many modern textbooks the number of practice problems is shrinking almost as if to imply that those who are unable to master the topic within the prescribed number of questions should consider themselves incapable of ever doing so. Best if they were to move on and try their hand with the next topic, or worse, consider mathematics as one of those areas that will forever remain inscrutable to them.

There are at least two possible reasons for this:

1. As the standardized curriculum grows, there is less time available to devote to each study unit. Lean manufacturing techniques seem to be permeating down into education.
2. As rote learning, a misnomer which will be discussed below, is removed from primary school education, the opportunity to reinforce learning with practice is also taken away from students who would otherwise greatly benefit.

But what if a student is capable of mastering a subject, but requires a great deal of reinforcement of the material in order to do so?

In order to fill the gap of what I felt was the diminishing availability of problem sets associated with each topic, the inspiration to generate dynamic worksheets struck and even more importantly making them available on the web. Naturally, this led me to seek out the use of TeX and LaTeX as the means by which to typeset mathematics worksheets effectively. Combining the two — that is generating problem sets dynamically and creating well typeset worksheets — will, I hope, tilt the balance back towards a reliance on practice, which for some students may mean much more than the median, and bring about a more inclusive approach to mathematics learning.

3 Rote vs. practice

It is important to distinguish from the onset a distinction between *rote* and *practice*. The reason being that rote learning is increasingly being considered superfluous in an age of technical wonderment. Why spend time memorizing anything when search engines are far more effective surrogates?

While rote and practice may be related in term of mechanics — they both rely on repetition — they differ in intent, and this difference strikes at the heart of the problem with tarnishing all forms of repetitious learning as rote. Rote implies tedium, or repetitious activity performed without purpose. Contrast that with practice, which is repetition done with the intent of gaining in capabilities. This in itself should be enough to show the wide gulf between the two.

However, there is a secondary aspect to the definition of practice which is that of gaining proficiency that leads us to consider just how we learn:

1. Do we first gain a kernel of comprehension when introduced to a topic, and then use practice

to reinforce that comprehension and achieve proficiency?

2. Or is it by the very act of practice that we get to even a basic level of comprehension? That is, does proficiency, even that acquired through mechanical repetition, lead then to comprehension?

It is often assumed, I feel, that the first approach is the only correct one.

When we reduce or remove the level of practice associated with learning a topic, we are adhering to this first approach, which may in fact be perfectly adequate for many students. However, I believe that there is a category of students for whom comprehension isn't so easy to come by through mere explanation. For these students, it is through the mechanical act of repetition that the topic reveals itself as patterns emerge.

Consider this as an example of that primary heuristic technique: that of trial and error.

When we denigrate all repetitious activity by labelling it as rote, we relegate this second important learning technique to the dustbin. As a consequence, students whose learning pattern mirrors the second approach are excluded from the learning process.

This distinction in approaches may be more easily recognized in the area of computer programming, where the relative numbers who fall into the two categories are reversed. It is a rare individual indeed who can proceed from learning a programming language or algorithm to immediate deep comprehension and proficiency. More often it requires the tentative steps of coding and stumbling with syntactic or semantic programming errors before the behaviour of the language and the program reveals itself. This trial and error method, along with isolating function to small digestible chunks is at the heart of software engineering. Should we not then elevate this learning approach to be on equal footing to the more linear one of elucidation leading to illumination?

4 The value of numeracy

If we accept that practice, even if in some cases this means much more practice than we might deem necessary, is essential for learning then what value does numeracy play towards the ultimate goal of mathematical literacy?

Calculators, which lost their novelty long ago, are now considered either relics of another earlier age or as commodities with little intrinsic value. There may be something to the concurrent decline in value of the once exotic calculator, and the value that we place on human numeracy. After all when a mechanical device that costs less than a fast-food meal is able to calculate numbers reliably and quickly, what need then of teaching numeracy to young students?

We are sometimes too quick at placing a value on a skill based solely on the economic cost needed to substitute for it. This mapping serves up, I believe, a grave fallacy when it comes to the learning of mathematics. Numeracy allows young children to play with and come to appreciate the behaviour of the most basic of mathematical elements, which are numbers. It gives them a chance to begin to recognize patterns and more importantly to gain a sense of mathematical intuition through a mastery of arithmetic.

Without this intuition, some students are unable to make the leap to symbolic manipulation as with algebra. But even before this, understanding fractions becomes difficult because unless one understands whole numbers how can one appreciate fractions of numbers and the delicate interplay that follows? And fractions are often a convenient entry into basic geometry. It is my assertion that once the foundation of numeracy is undermined, we make the resulting edifice of mathematical literacy shaky at best, and for some students impossible to construct.

Equally important, but in an entirely different plane, is that it takes away an important avenue through which young children are able to express mastery over their environment. Like mastery over language, basic numeracy allows them to interact with, recognize patterns within, give a name to, and to apply groupings to the physical world around them. The value of this should not be overlooked. It is an important aspect of giving a child self-confidence in their own ability to express their independent judgements.

5 Method

If one accepts the motivation outlined in the previous sections, the question remains, "How best to offer up practice when motivation for it is limited in a school setting?" This is the question that confronted me when I was trying to come up with learning materials for my daughter.

A search for mathematics worksheets on the web revealed an emphasis on interactive presentations, with insufficient variation in the problems themselves to fulfil my goal of increasing the level of practice in my daughter's study. Other sites had limited topics and very basic levels of difficulty.

From this search, I sought to construct worksheets that fulfilled the following design criteria:

1. The list of worksheets should be easily extensible. That is, if the worksheets were to be

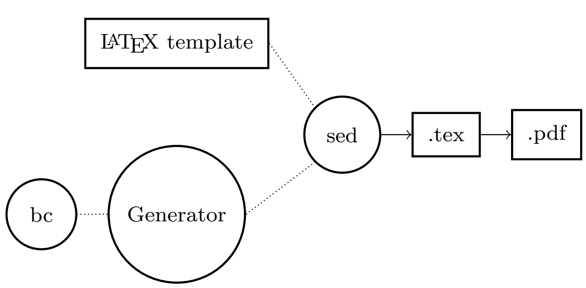
offered up on a web site, which was a stated goal, then it should be relatively easy to add to their numbers without having to affect the web site programming. This implied a database driven approach to cataloguing and invoking worksheets.

2. Parameterized worksheet generation. Being dynamic, the worksheet generators should be able to generate different groupings by merely specifying parameters. To put this in terms of concrete examples, multiplication worksheets could be limited to generate problems dealing with, for instance, multiplication tables between 1–4, 5–8, 9–12, or any specified range. Or in the case of geometry, worksheets on recognizing angles could be limited with the use of parameters to showcase only certain types of angles, i.e., only acute and obtuse.
3. Graduated levels of difficulty. An essential component of each worksheet generator was that not only would it allow for different operands, but that it should take the student from the basics of a topic gradually through more difficult aspects of the same topic.
4. The ability to create beautiful documents. This final criteria led me naturally to the doorstep of TEX and LATEX.

Given this desired feature set, any number of programming languages and frameworks could have been chosen. Ruby on Rails was selected only because the author had some experience with Ruby, the language. This provided the necessary components to put the application on the web.

In order to animate the application, basic Unix tools were used: `bash`, `sed`, `bc`, `cron`. Development work was done under Ubuntu, and the application then deployed on a Sun Microsystems SPARCserver running Solaris 10. If nothing else, it showed the extent and ease with which frameworks and TEX Live accommodate cross-platform development.

The diagram below illustrates the architecture:



Each group of worksheets that are related by topic have a `bash` script associated with them. The script utilizes the built-in random number generator to generate, and `bc` to then normalize the operands. The operands are substituted into the base LATEX template for that group with the use of `sed` to generate a spooled `.tex` file. This file is compiled into a `.pdf` file which is served back to the client browser. The invocation command, along with the associated parameters for a given worksheet at a given level of difficulty are stored in a database, as are the counters used to instantiate the spooled `.tex` file.

6 Conclusions

The value of both numeracy and practice in the teaching of grade school mathematics has been presented in this paper. The discussion has also touched upon the larger question of just how children learn. It is my hope that this work will encourage those children who may struggle with the subject to look upon it not as an insurmountable mountain, but only as a road less travelled, one that they have the privilege of walking along and in turn teaching us, by their example, just what learning really means.

7 Acknowledgements

The name of the web site, `www.bansisworld.org`, takes its inspiration from that of my late maternal grandfather, Prof. Bansi Lal, Professor Emeritus of the Department of Mathematics at D.A.V. College, Jalandhar, India. He spent much of his career focused on the teaching of mathematics, and it is my hope that Bansi's World in this modern web context will become synonymous with the earlier, original incantation of "Bansi Lal's world", which intimated the idea of open mathematics education.

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